

## SOME CONSIDERATIONS REGARDING THE EFFICIENCY OF THE ELECTROMECHANICAL MOTION

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Abstract: Control techniques for servo drive which run at variable speed for prolonged time is developed on the base of minimum energy dissipation in a feed-forward structure. The optimal control laws are determinate using the estimated values of the main perturbation - the load torque. Different aspects of the electromechanical motion efficiency are presented regarding the influence of the desired time of execution, the shape of trajectory and the last torque.

Keywords: Motion control, feed-forward control, energy saving, efficiency of dynamical drive system

### 1. INTRODUCTION

The performance of electromechanical systems depends on numerous variables such as the mechanical design the operating environment and the control system. The control system must perform functions such as positioning, trajectory tracking, suppression of vibration, disturbance rejection. The important point to note is that the commands used the perform a desired motion have a variety of shapes. As will see, the shape of the commands can greatly affect system performance and must be treated as a design variable rather than a given variable.

In the most sophisticated mechanical system such as a humanoid robot the motion control requires two different coordinate systems: joint space and external (e.g., Cartesian) space, witch is needed to reference a task to the external world. The kinematics variables, e.g., position, velocity and accelerations are converted to motor commands. The inverse dynamics model receives desired position, velocity and acceleration commands  $\alpha_d, \dot{\alpha}_d, \ddot{\alpha}_d$  and computes the appropriate feed forward commands  $\mathbf{u}_{ff}$  for drive control system (fig.1).

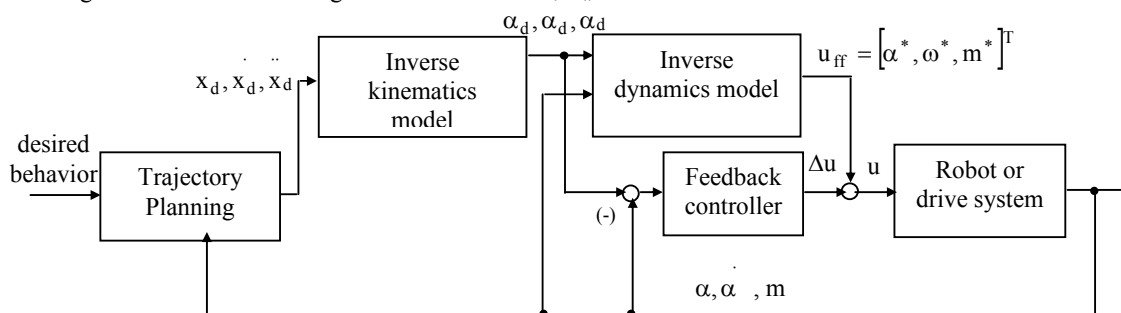


Fig.1 Control using the inverse kinematics and inverse dynamics models

The inverse kinematics model is used to create a control policy that is based on Cartesian state. The

Cartesian information about the target  $X_d, \dot{X}_d, \ddot{X}_d$  is sent to the inverse kinematics model and converted to desired joint angles trajectories  $\alpha_d, \dot{\alpha}_d, \ddot{\alpha}_d$ .

In fig.1 the feed back controllers read the desired position, velocity and feed forward commands, i.e.,

$\alpha_d, \dot{\alpha}_d, \ddot{\alpha}_d, u_{ff}$  at each degree-of-freedom and compute the PIDFF (proportional, integral derivative and feed forward) command  $u$ .

Our work is restricted to one degree of freedom movement. Complex movement can be achieved by superimposing many such basic models. We don't focus on the classic kinematics problem, that is, on the transformation from kinematics plans in external coordinates (the actuator space in witch motor command must be issued).

#### 1. FINDING THE OPTIMAL CONTROL LOW

We consider the inverse dynamics model from the point of view of energetic efficiency. Let be  $\alpha_f$  the final desired position and  $m_L$  the load torque of drive system. The efficiency of motion is:

$$(1) \eta = \frac{\int_{t_0}^{T_f} m_L \cdot \omega dt}{\int_{t_0}^{T_f} m_L \cdot \omega dt + \int_{t_0}^{T_f} R \cdot i^2 dt + \int_{t_0}^{T_f} p_c dt}$$

where  $T_f$  is the final time of motion, load dependent losses are  $\int_{t_0}^{T_f} R \cdot i^2 dt$  and no-load losses are denoted

by  $\int_{t_0}^{T_f} p_c dt$ .

We neglected the stray-load losses witch arise from harmonics and circulating currents. At-load and no-load losses make up roughly 2/3 and 1/3 of the total losses, respectively. Percentages vary somewhat, depending on the manufacturer and motor.

Let consider the no-load losses

$$p_c = \chi \cdot R \cdot I_N^2$$

where  $\chi \leq 1$  and  $R \cdot I_N^2$  are rated load-dependant losses.

The objective of the inverse dynamics model can be stated as follows:

Given the specifications of the drive system behavior, design a feed-forward control law  $u_{ff}$  such that the feedback controllers exhibit the desired behavior with maximum efficiency.

We consider that the load torque  $m_L$  will be estimated by iterations for  $t_k < t < t_{k+1}$ ,  $k = 1, n$ . The estimated load torque  $m_{LK}$  is constant for each iteration and

$$(2) \int_{t_k}^{t_{k+1}} m_{LK} \cdot \omega dt = m_{LK} \cdot \Delta\alpha_k$$

with

$$(3) \Delta\alpha_k = \alpha_{k+1} - \alpha_k$$

The efficiency (1) for  $t_k < t < t_{k+1}$  is

$$(4) \eta = \frac{m_{LK} \cdot \Delta\alpha_k}{(m_{LK} \cdot \Delta\alpha_k + \int_{t_k}^{t_{k+1}} R \cdot i^2 dt + \int_{t_k}^{t_{k+1}} p_c dt)}$$

For maximization of (4) it must to minimize the performance index

$$(5) I_k = \frac{1}{m_{LK} \cdot \Delta\alpha_k} \int_{t_k}^{t_{k+1}} R \cdot i^2 dt$$

In the following we consider the motion of a simple one coordinate (one degree of freedom) electromechanical system described by the equations:

$$(6a) m = m_L + J \frac{d\omega}{dt}$$

$$(6b) \alpha = \omega$$

where  $J$  is the total moment inertia,  $\omega$  the speed and  $\alpha$  the position.

We consider also the in built motor feed-back control for torque and on constant motor flux. The motor torque is :

$$(7) m = Ki$$

where  $K$  is the torque constant.

There seems to be general agreement that the most effective control scheme for drives is an cascaded or nested structure with a fast inner torque loop, to which outers speed and position loops are superimposed. The feed-forward control is:

$$(8) u_{ff} = [m^*, \omega^*, \alpha^*]^T$$

Taking in consideration that the inner torque loop is fast, we consider

$$(9) m^* = m$$

Under this conditions from (5) and (7) results:

$$(10) I_K = \frac{R}{K^2 \cdot m_{LK} \cdot \alpha_K} \int_{t_K}^{t_{K+1}} m^2 dt$$

The optimal trajectory control law  $m_K^*$  which can transfer the electromechanical system (6a) from initial state at  $t = t_K$  to the final state at  $t = t_{K+1}$  with minimum cost can be obtained using Hamilton's principle.

The Hamiltonian is:

$$(11) H_K = \frac{R \cdot m^2}{K^2 \cdot m_{LK} \cdot \alpha_K} + \lambda_1(m - m_{LK})/J + \lambda_2 \cdot \omega$$

The costate equations are:

$$(12) -\dot{\lambda}_1 = \frac{\partial H_K}{\partial \omega} = \lambda_2$$

$$(13) -\dot{\lambda}_2 = \frac{\partial H_K}{\partial \alpha} = 0$$

The stationary condition is:

$$(14) 0 = \frac{\partial H}{\partial u} = \frac{2 \cdot R \cdot m}{K^2 \cdot m_{LK} \cdot \alpha_K} + \frac{\lambda_1}{J}$$

From equation (12, 13) results:

$$(15) m = m^* = At + B \quad t_K < t < t_{K+1}, \quad K = \overline{1, n}$$

with A and B constants which have to be determinate from initial and final conditions.

The model of the feed-forward law (15) is represented in fig.2. The corresponding discrete equation is

$$(16) m^*(k+1) = m^*(k) + A \cdot t$$

with t as sampling period and the initial condition

$$(17) m^* = B$$

The constant A and B depends on the estimated last torque at k iteration and on the global initial and final conditions. Because the control law is linear it is possible to determine the constants A and B superposition the global condition marking  $m_{LK} =$  constant and then adjusting the load torque at each iteration with estimated values.

The global initial and final conditions are:

$$\text{for } t = 0 \quad \omega = \omega_0 \quad \alpha = \alpha_0$$

$$(18)$$

$$\text{for } t = T_f \quad \omega = 0 \quad \alpha = \alpha_f$$

Where  $T_f, \alpha_f$  are desired final time and final position of motion.

From motion equation (6a) we have

$$(19) J\omega = A \frac{t^2}{2} + B \cdot t - m_L \cdot t + C, \quad C = J\omega_0$$

$$(20) \alpha = \frac{1}{2} \left[ A \frac{t^3}{6} + (B - m_L) \cdot t + J \cdot \omega_0 \cdot t + D \right], \quad D = \alpha_0$$

Using (18, 19, 20) we obtain:

$$(21) m^* = -2 \cdot \left( m_d + \frac{m_{d1}}{2} + m_{d2} \right) \frac{t}{T_f} + m_d - \frac{2}{3} m_{d1} - m_{d2} + m_L$$

with:

$$(22) m_d = \frac{6J\alpha_f}{T_f^2}, \quad m_{d1} = \frac{6J\omega_0}{T_f}, \quad m_{d2} = \frac{6J\alpha_0}{T_f^2}$$

From (21) result the discreet command law

$$(23) m^*(k+1) = m^*(k) - 2 \cdot (m_d + \frac{m_{d1}}{2} + m_{d2}) \cdot t$$

$$(24) m^*(k) = m_d - \frac{2}{3} m_{d1} - m_{d2} + \omega_{LK}$$

The algorithm for complete command law is

$$m_{d0} = \frac{6J\alpha_f}{T_f^2}, \quad m_{d10} = \frac{6J\omega_0}{T_f}, \quad m_{d20} = \frac{6J\alpha_0}{T_f^2},$$

$$\hat{m}_{L0} = m_N \quad \text{for } k=1 \text{ repeat}$$

$$m^*(k+1) = m^*(k) - 2 \cdot (m_d + \frac{m_{d1}}{2} + m_{d2}) \cdot t$$

$$(25) \omega^*(k+1) = \omega^*(k) + (m^*(k+1) - \hat{m}_L(k) \cdot t) / J$$

$$\alpha^*(k+1) = \alpha^*(k) + \omega(k+1) \cdot t$$

Load torque estimator

$$\hat{m}_L(k+1) = \text{load\_torque\_estimator}();$$

//Load torque compensation

$$m^*(k) = m^*(k+1) + \hat{m}_L(k+1) - \hat{m}_L(k);$$

$$\omega^*(k) = \omega^*(k+1);$$

$$\alpha^*(k) = \alpha^*(k+1);$$

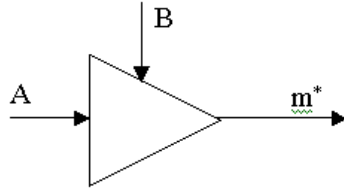


Fig. 2 The first order torque command low

The estimation of the load torque is based on the speed and current measurement and on the total moment of inertia J.

### 3. THE SYSTEM PERFORMANCE EVALUATION

We can directly evaluate the performance of the feed-forward control (21) under condition (9). After that, using a real drive system with cascaded feedback controllers we implemented feed-forward control (25) for determination of the efficiency in real condition that is taking into account the limited time response of the torque loop.

The efficiency of the motion which execute  $\alpha_f$  (rad) in  $T_f$  (seconds) with a constant load torque  $m_L$  is:

$$(26) \eta = \frac{m_L \cdot \alpha_f}{m_L \cdot \alpha_f + \frac{12 \cdot R \cdot \alpha_f^2 \cdot J^2}{K^2 \cdot T_f^3} + \frac{R \cdot m_L^2 \cdot T_f}{K^2} + \chi \cdot R \cdot I_N^2 \cdot T_f}$$

This result corresponds to command low (21) and  $\alpha_0 = 0, \omega_0 = 0$ ; The maximum efficiency results for

$$(27) T_f^2 = \frac{6\alpha_f J}{m_L}$$

with

$$(28) R = \gamma \frac{P_N}{I_N^2}$$

$$\gamma = \frac{1-\eta}{(1+\chi) \cdot \eta_N}; m_L = \beta \cdot m_N$$

the efficiently (26) takes the form

$$(29) \eta_2 = \frac{\beta \cdot m_N \cdot \alpha_f}{\beta \cdot m_N \cdot \alpha_f + \frac{\gamma \cdot P_N \cdot m_d^2 \cdot T_f}{3 \cdot m_N^2} + \gamma \cdot P_N \cdot \beta^2 \cdot T_f + \chi \cdot \gamma \cdot P_N \cdot T_f}$$

For efficiently comparison, we adopt another control low ,

$$(30) m^* = \begin{cases} \frac{3}{2} m_d + m_L & \text{for } 0 < t \leq \frac{T_f}{2} \\ -\frac{3}{2} m_d + m_L & \text{for } \frac{T_f}{2} < t \leq T_f \end{cases}$$

Which produce the same displacement

$\alpha_f$  in the same final time  $T_f$ . In this case efficiency is

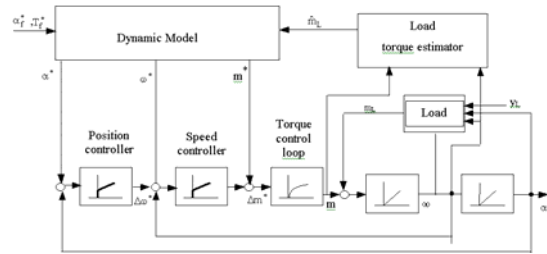


Fig.3 Position control system with feed-forward from a dynamic model which calculates the desired trajectory of the drive

$$(31) \eta = \frac{m_L \cdot \alpha_f}{m_L \cdot \alpha_f + \frac{16 \cdot R \cdot \alpha_f^2 \cdot J^2}{K^2 \cdot T_f^3} + \frac{R \cdot m_L^2 \cdot T_f}{K^2} + \chi \cdot R \cdot I_N^2 \cdot T_f}$$

From (26,30) result that in no load operation ( $m_L = 0$ ) the copper losses are 25% gather with controls low (30) than with optimal control low (26).

The relations obtained for efficiency are valid for point- to- point control under maximum velocity , that is the maximum velocity is reached before the motor moves half to the target position .

Two levels of optimal motion control are available:

- Optimal trajectories (25) but sub optimal time of execution (27), that is:

$$T_f^2 \neq \frac{6 \cdot J \cdot \alpha_f}{m_L}$$

- Optimal trajectories (25) and optimal time of execution (27).

When the motion time gets shorter a penalty power dissipation factor must be taken in account.

#### 4. EXPERIMENTAL RESULTS

The experimental result was carried out with the position controller servo drive represented in fig. 3. It is possible to implement different control laws in aim to compare the efficiency of motion. First we tasted the performance of load torque estimator, which is essential for maintaining the optimal efficiency under load torque variation.

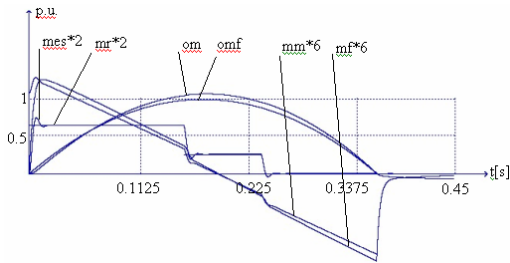


Fig.4 The optimal control law under the load torque variation and time of execution shorter than optimal time ( $T_f \neq T_{optim}$ ;  $\eta=0.25$ )

Figure 4 shows the capability of torque estimator to detect the step variations of the load disturbances. By compensating the load disturbances the optimal trajectory of motion remain practically unchanged.

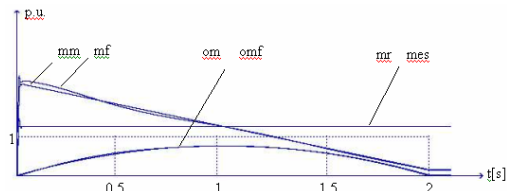


Fig.5 The optimal control law and optimal time of execution;  $\eta=0.82$

In figure 5 was represented the optimal trajectories when time of execution is optimum. It seems that there are many features that have been observed in human and primate arm movements that is closely related to the minimum energy criteria trajectories.

In fig.6 and 7 are represented the responses of servo drive at two commands laws: the movements with the optimal control law (25) and the primate and human arms movements.

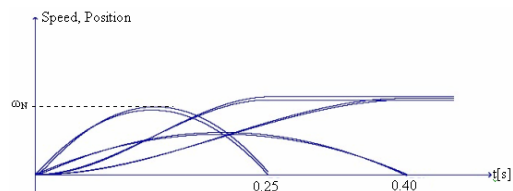


Fig.6 Position and velocity for two movements with different final times under optimal control law

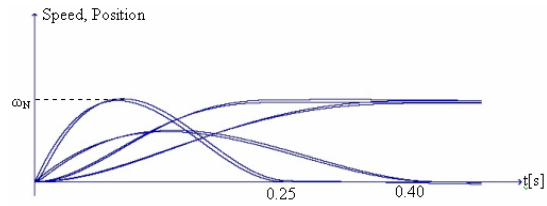


Fig.7 Position and velocity for two movements with different final times observed in human and primate arms

Another conclusion depicted from fig. 6 and 7 is the following: the same motor, with the same load, realize the same final position with deferent's efficiency if the time of execution is not the same.

The optimal control law was obtained under assumption that  $m^*=m$ , that is neglecting the response times of torque loop. This approximation affect the efficiency if the feedback controller has not the capability to follows as well as possible the time-varying optimal trajectories. In the figures 6 and 7 the tracking of optimal torque and velocity profiles are quiet well and efficiency is very closed to the optimal values.

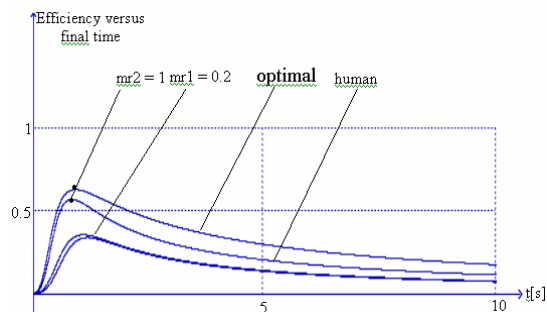


Fig.8 The efficiency of motion versus final desired time for two different load torque and two controls laws: optimal and human observed arms movement

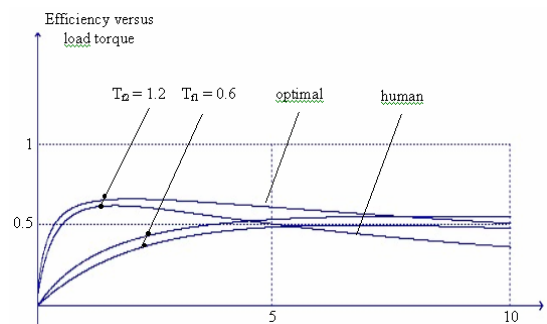


Fig.9 The efficiency of motion versus load torque for two deferent's execution times and two controls laws: optimal and human observed arm movement

In figures 8 and 9 was represented the efficiencies in two cases: with optimal control low and in humanoid arm movement case. If the load torque decrease it must increase the time of execution for maintaining maximum efficiency. Figure 10 shows the total energy dissipation versus motion time  $T_f$ .

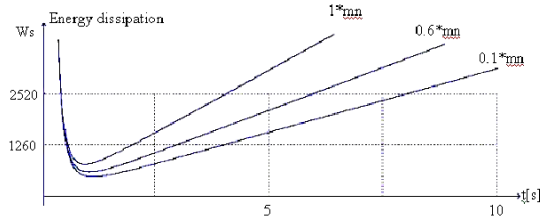


Fig.10 The optimal dissipation versus motion time  $T_f$  for three different load torque.

## 5. CONCLUSIONS

Efficiency is critical to long-term economical operation of electrical drives. A point of perspective is that less than 3% of a motors life-cycle cost represents purchase price and installation costs- the rest goes to electricity charges. Even modest efficiency improvements yield substantial energy savings.

In the paper the following main problem was analyzed: move the load by required distance and accuracy within time  $T_f$ , while minimizing power dissipation. Control low for selecting the best torque and velocity profile that results in minimum energy dissipation within motion time was elaborated. A C++ software package to allow creating dynamics simulation of drive system with optimal development time was elaborated.

The software package offers the following development features:

- Motor control loops for PIDFF and inverse dynamics control (fig.1);
- Load torque estimator;
- Easy data collection during control and drive performances computation;
- Very easy to extend for user-specific motor control problems;

These properties of software package make it well suited for applications in the areas like:

- Special drive systems and robot control research and education.
- Studies of motor control from the viewpoint of efficiency in rhythmic and discrete movement and studies of motor learning.
- Energy saving education.

The efficiency depends strongly of the imposed time of motion and of the last torque. The

specification of the desired behaviors of the closed loop system is usually formulated in terms of qualitative performance including stability, accuracy, response speed and robustness. These specifications must be completed with the shape of the command, which perform the desired movement task and the time of execution. Choosing the fastest response execution time for a desired movement don't represent the best solution from the point of view of the costs. There are many opportunities when the fast time of execution is not necessary. In these cases (especially for heavy-load motion control), important savings of energy costs are obtained by imposing the shape of trajectory and the optimal time of execution. The trajectory planning (fig.1) of the desired system behavior takes in account the energy efficiency of the motion. The optimal time of execution depends also on the last torque profile. It is possible to adopt the execution time nearest to the optimal if we know apriority the last torque profile. That is possible for example with a learning load torque system and an on-line torque estimator for corrections of the off-line learning torque profile. All the experimental results illustrated in paper was carried out with an 45KW, 440 DC motor.

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